EFFECT OF FLOW TWISTING ON HYDRAULIC RESISTANCE AND HEAT EXCHANGE

V. Ya. Suslov and N. A. Makarov

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On the basis of dimensional analysis through a differentiated approach to the dimensions of length we have obtained formulas for the effect of flow twisting in a circular tube on the hydraulic resistance and exchange of heat.

As flow twisting finds increasing application for purposes of intensifying heat exchange, we note an increasing need for reliable information regarding its effect on hydraulic resistance and heat exchange. Presently this effect is described by means of empirical relationships that differ substantially in terms of results and are not in sufficient agreement with the statements of dimensional analysis.

Based on dimensional analysis, in this paper we have generalized familiar criterial relationships for uniform motion in a tube to the case of twisted motion. On the basis of the recommendations of Huntley [1] we have assigned his units of measurement to each of three dimensions of length over the directions of the coordinate axes. Such an increase in comparison with the traditional approach in the number of basic measurement units leads to single-valued relationships between hydraulic resistance and heat exchange and the degree of twisting without resort to experimental data.

Let us assume that the axis of the tube coincides with the x axis of a rectangular coordinate system xyz. With uniform flow, the pressure losses per unit length of tube, i.e., dp/dx, are determined by the average velocity of motion U of the tube, the tube diameter D, and by the density and viscosity of the liquid, ρ and μ, respectively. In the case of flow twisting, the average circular component of flow velocity W is added to the number of quantities which affect hydraulic resistance. Its direct application leads to the appearance in the criterial equation of a cofactor which when W = 0 vanishes or becomes infinite, which contradicts physical reality and prevents us from using the familiar results from uniform flow to generalize to the case of a flow with twisting. Therefore, in the following we will use the resulting velocity $V = \sqrt{U^2 + W^2} = U/\cos\varphi = W/\sin\varphi$, where φ is the angle between the velocity vectors U and V.

Let us present the pressure gradient as a function of the remaining quantities in the form

$$\frac{dp}{dx} = CD^a U^b \rho^c \mu^d V^e. \qquad (1)$$

The dimension $[U] = L_x T^{-1}$, $[\rho] = ML_x^{-1}L_y^{-1}L_z^{-1}$, $[dp/dx] = ML_y^{-1}L_z^{-1}T^{-2}$. Here L_x, L_y, and L_z are units of length measurements in the direction of the axes x, y, and z; M and T are units of mass and time measurement. When we take into consideration the axial symmetry of the process relative to the x axis [1], the dimension $[D] = L_y^{1/2}L_z^{1/2}$.

Let us determine the dimension of velocity V. It is obvious that when $\varphi = 45°$ the dimension of V with identical exponents must include L_x and L_w which are the units of length measurement for the circumference in the yoz plane: $[V] = L_x^{1/2}L_w^{1/2}T^{-1} = L_x^{1/2}L_y^{1/4}L_z^{1/4}T^{-1}$. When $\varphi = 0°$, $[V] = L_x T^{-1}$, and when $\varphi = 90°$, $[V] = L_w T^{-1} = L_y^{1/2}L_z^{1/2}T^{-1}$. We can see from these expressions that the exponents for the units of length in the dimension of V are linearly dependent on φ. Therefore, in the general case $[V] = L_x^{1-2\varphi/\pi}L_y^{\varphi/\pi}L_z^{\varphi/\pi}T^{-1}$.

According to [1], the length units in the dimensional formulas for viscosity and flow velocity coincide:

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$$[\mu] = L_x^{-(1-2\varphi/\pi)} L_y^{-\varphi/\pi} L_z^{-\varphi/\pi} MT^{-1}.$$

Having substituted into (1) the dimensions of the corresponding quantities, we obtain

$$ML_y^{-1}L_z^{-1}T^{-2} = (L_y^{1/2}L_z^{1/2})^a (L_xT^{-1})^b (ML_x^{-1}L_y^{-1}L_z^{-1})^c \times \times (L_x^{-\alpha}L_y^{-\beta}L_z^{-\beta}MT^{-1})^d (L_x^{\alpha} L_y^{\beta} L_z^{\beta} T^{-1})^e, \tag{2}$$

where $\alpha = 1 - 2\varphi/\pi$, $\beta = \varphi/\pi$.

Having equated the exponents for the units of length, time, and mass, we obtain the following system of equations:

$$0 = b - c - \alpha d + \alpha e, \quad -1 = \frac{a}{2} - c - \beta d + \beta e, \quad -1 = \frac{a}{2} - c - \beta d + \beta e, \quad -2 = -b - d - e, \quad 1 = c + d, \tag{3}$$

from which it follows that $a = -1 - d$, $b = 2 - d - (1 - \alpha d)/(1 - \alpha)$, $c = 1 - d$, $e = (1 - \alpha d)/(1 - \alpha)$.

The coefficient of hydraulic resistance

$$\lambda = \frac{dp/dx}{\rho U^2/2D} = 2C\,\mathrm{Re}^{-d} (\cos\varphi)^{-\frac{1-\alpha d}{1-\alpha}}. \tag{4}$$

For uniform turbulent flow, if we use the Blasius formula $\lambda_0 = 0.316\mathrm{Re}^{-0.25}$, we obtain

$$\lambda/\lambda_0 = (\cos\varphi)^{-\frac{3\pi}{8\varphi} - \frac{1}{4}}. \tag{5}$$

The coefficient of heat conduction α_h for the case of uniform flow in a tube is determined by the quantities μ, ρ, D, and U, by the coefficient of thermal conductivity λ_h, and by the heat capacity C_m per unit liquid mass. The resulting velocity V in the twisting of a flow is added to the number of quantities affecting heat exchange.

Let us present the relationship between the coefficient of heat conduction and the remaining quantities in the form

$$\alpha_h = C\mu^a \rho^b D^c \lambda_h^d C_m^e U^f V^g. \tag{6}$$

According to [1], the dimension of heat capacity $[C_m] = L_x^{2/3}L_y^{2/3}L_z^{2/3}T^{-2}\Theta^{-1}$, where Θ is the unit of temperature. Proceeding from the definition of the concept of the coefficients of heat conduction and thermal conductivity, we find that $[\alpha_h] = L_x^{-1/3}L_y^{1/6}L_z^{1/6}\cdot MT^{-3}\Theta^{-1}$, $[\lambda_h] = L_x^{-1/3}L_y^{2/3}L_z^{2/3}MT^{-3}\Theta^{-1}$.

Having substituted the dimensions of the corresponding quantities into (6), we have

$$L_x^{-1/3}L_y^{1/6}L_z^{1/6}MT^{-3}\Theta^{-1} = (L_x^{-\alpha}L_y^{-\beta}L_z^{-\beta}MT^{-1})^a \times \times (ML_x^{-1}L_y^{-1}L_z^{-1})^b (L_y^{1/2}L_z^{1/2})^c (L_x^{-1/3}L_y^{2/3}L_z^{2/3}MT^{-3}\Theta^{-1})^d \times \times (L_x^{2/3}L_y^{2/3}L_z^{2/3}T^{-2}\Theta^{-1})^e (L_xT^{-1})^f (L_x^{\alpha}L_y^{\beta}L_z^{\beta}T^{-1})^q. \tag{7}$$

Having equated the exponents for identical units of measurement, we obtain the system of equations:

$$-\frac{1}{3} = -\alpha a - b - \frac{d}{3} + \frac{2}{3}e + f + \alpha g, \quad \frac{1}{6} = -\beta a - b + \frac{c}{2} + + \frac{2}{3}d + \frac{2}{3}e + \beta g, \quad \frac{1}{6} = -\beta a - b + \frac{c}{2} + \frac{2}{3}d + \frac{2}{3}e + \beta g, \quad 1 = a + b + d, \quad -3 = -a - 3d - 2e - f - g, \quad -1 = -d - e, \tag{8}$$

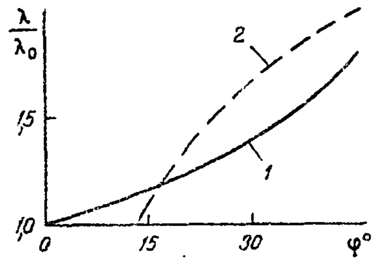

Fig. 1

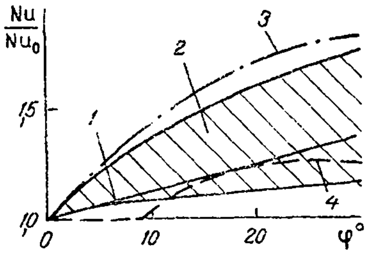

Fig. 2

Fig. 1. The ratio λ/λ_0 as a function of the flow twisting angle φ: 1) formula (5); 2) empirical relationship [2].

Fig. 2. Ratio Nu/Nu_0 as a function of the flow twisting angle φ: 1) formula (10); 2) region over which the experimental data of various authors are scattered in the twisting of a flow by a twisted strip [4]; 3, 4) empirical relationships [3, 2].

from which we find that $a = -b + e$, $c = b - 1$, $d = 1 - e$, $f = b(1 - \alpha/(1 - \alpha)) - e$, $g = \alpha b/(1 - \alpha) + e$.

It follows from expression (6) that

$$\frac{\alpha_h D}{\lambda_h} = Nu = C Re^b Pr^e (\cos\varphi)^{1-\frac{\pi}{2\varphi}-e}. \tag{9}$$

Bearing in mind that with uniform turbulent flow $Nu_0 = 0.023 Re^{0.8} Pr^{0.4}$, we have

$$Nu/Nu_0 = (\cos\varphi)^{0,6-\frac{\pi}{2\varphi}}. \tag{10}$$

Let us compare the results of the calculations by means of formulas (5) and (10) with the experimental data presented in [2-4]. The degree of flow twisting was characterized in [2, 3] by the ratio of the circular and longitudinal momenta of the liquid. In order to make the transition from this quantity to the angle φ we resorted to the empirical data of [2]. The results from various authors, dealing with the twisting of a flow by means of a twisted strip, are represented in [4]. With the transition from the strip-twisting parameter to the angle φ it was assumed that the angle of flow twisting at each radius is equal to the corresponding angle of strip twisting.

As we can see from Figs. 1 and 2, the results from the calculations according to formulas (5) and (10) are in satisfactory agreement with the experimental data.

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